FREQUENCY CHARACTERISTICS OF SAMPLED-DATA CONTROL SYSTEMS

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Abstract. For the description of linear digital-analogous control systems normally discrete transfer functions and discrete frequency characteristics are used. If the frequencies of the input signal have rational relation with the period T_s of the system then experimental determination of the discrete frequency response is possible. But in practice it is very difficult to accomplish this conditions which causes unstable experimental results.
With the concept of the parametric transfer function it is possible to generalize the discrete frequency response to the parametrical frequency response (PFR). This function $W(j\nu, t)$ depends on frequency ν and time delay t and is periodical with respect to the time delay:

$$W(j\nu, t) = W(j\nu, t + T_s) \quad .$$

Here the period of the system is denoted by T_s. Furthermore, for the time-instants $t = 0, T_s, 2T_s, \ldots$ the PFR is equal to the usual discrete frequency response.
With the help of the PFR it is possible to investigate linear digital-analogous systems for any time t in $[0, T_s]$ and all frequencies ν. In this article the theoretical approach and some methods of the experimental determination of the PFR are presented.
Furthermore we will present experimental results received by this methods in the lecture.

Key Words. Sampled-data system, parametric transfer function, frequency response, digital control.

1 Introduction

A method for analysing a linear time-invariant system is the experimental investigation of its frequency response. This method is practicable for relativly simple systems. The discrete frequency response $W_d(j\nu) = G(e^{j\nu})$ describes the reaction of the system at the sampling points $t = 0, T_s, 2T_s, \ldots$. If we consider analog-digital systems, the frequencies of the input signal must have also rational relationship to the sampling frequency. This condition leads to unstable experimental results, because the harmonic oscillation generator and the digital converter are working not exactly ideal. So the interpretation of the experimental results is sophisticated.

The concept of the parametric transfer function (PTF) allows a new approach. The PTF is a generalization of the transfer function to linear time-periodic systems, which include the sampled-data systems. With the PTF we can define the parametric frequency response (PFR). This PFR is measurable for all frequencies. In this paper a method is presented, which bases on the harmonic analysis of the system response to sinusoidal and cosinusoidal input signals.

For small frequencies $\nu << \omega = \frac{2\pi}{T_s}$ the PFR is approximative to the discrete frequency response $W(j\nu, t) \approx W_d(j\nu)$ and $W(j\nu, t)$ becomes time-invariant. In other cases this condition is not fulfilled and the PFR is essential unstationary, which yields to errors in the frequency analysis.

2 Parametric Transfer Functions

Let us have the input signal $x(t)$ and the output signal $y(t)$. Now we assume, that the connection between these two signals is given by a linear operator L

$$y(t) = L[x(t)] \quad .$$

Then we can define the parametric transfer function $W(s,t)$ by

$$W(s,t) = L[e^{st}]e^{-st}$$

with the complex variable s and the time t. If we have for example the integral operator

$$L[x(t)] = \int_0^t x(\tau)\,d\tau \tag{1}$$

we get the PTF

$$\begin{aligned} W(s,t) &= e^{-st}\int_0^t e^{s\tau}\,d\tau \\ &= \frac{1-e^{-st}}{s} \quad . \end{aligned} \tag{2}$$

This example yields also the the ordinary transfer function $\frac{1}{s}$ for an integrator, if t tends to infinity.

It can be shown, that the PTF of time-invariant systems is equal to the ordinary transfer function. For periodic operators with the period T_s

$$L[x(t+T_s)] = L[x(t)]$$

the PTF is periodic in t with the same period

$$W(s,t) = W(s,t+T_s) \quad .$$

Furthermore we get

$$W(s,t) = W_1(s,t) + W_2(s,t)$$

for parallel operators with the PTF $W_1(s,t)$ and $W_2(s,t)$, and

$$W(s,t) = W_1(s)W_2(s,t)$$

for a connection of an linear time-invariant operator with the transfer function $W_1(s)$ and an linear operator with the transfer function $W_2(s,t)$.

3 Parametric Frequency Response

Similar to the ordinary frequency response we define the parametric frequency response as the transfer function on the imaginary axis and split it into an real and imaginary part:

$$W(j\nu,t) = P(\nu,t) + jQ(\nu,t), \quad t \in [0,T_s] \quad . \tag{3}$$

The real and imaginary parts of the PFR are periodical with respect to the parameter t:

$$\begin{aligned} P(\nu,t) &= P(\nu,t+T_s) \\ Q(\nu,t) &= Q(\nu,t+T_s) \quad . \end{aligned}$$

The response of the system to the input signal $x_s(t) = \sin(\nu t)$ has the form

$$y_s(t) = P(\nu,t)\sin(\nu t) + Q(\nu,t)\cos(\nu t) \tag{4}$$

and for $x_c(t) = \cos(\nu t)$ it has the form

$$y_c(t) = P(\nu,t)\cos(\nu t) - Q(\nu,t)\sin(\nu t) \quad . \tag{5}$$

According to equation (4) and (5) we get the equations

$$P(\nu,t) = y_s(t)\sin(\nu t) + y_c(t)\cos(\nu t) \tag{6}$$

$$Q(\nu,t) = y_s(t)\cos(\nu t) - y_c(t)\sin(\nu t) \quad . \tag{7}$$

These two equations are describing a method for experimental determination of the PFR out of the steady parts of the responses $y_s(t)$ and $y_c(t)$ for a fixed frequency ν. After a time in which the transient parts of the responses are practically vanished we measure $y_s(t)$ and $y_c(t)$ for $NT < t < NT + T_s$ with the period $T = \frac{2\pi}{\nu}$ of the input signal. The time NT after which the transient part vanishes depends on the system, for the interesting frequencies $N > 10$ is necessary.

Sometimes the frequencies of the system and the input signal have a rational relation. This means there exists natural numbers r, m with

$$\frac{\omega}{\nu} = \frac{T}{T_s} = \frac{r}{m} \quad . \tag{8}$$

Because of mathematical reasons we have to exclude frequencies for which

$$\frac{2r}{m}$$

is an integer. For these cases S.Volovodov suggested a method for experimental determination of the PFR, which only needs one measuring for every frequency ω. Because $P(\nu,t)$ and $Q(\nu,t)$ are periodical, we can generate their Fourier-series. From a practical point of view we only need a finite number of summands, so we get:

$$\begin{aligned} P(\nu,t) &\approx \sum_{k=0}^{n} A_k(\nu)\sin(k\omega t) \\ &\quad + B_k(\nu)\cos(k\omega t) \\ Q(\nu,t) &\approx \sum_{k=0}^{n} C_k(\nu)\sin(k\omega t) \\ &\quad + D_k(\nu)\cos(k\omega t)\,. \end{aligned} \tag{9}$$

Together with formula (4) we obtain:

$$\begin{aligned} y_s(t) &= \sum_{k=0}^{n} \Big[A_k(\nu)\sin(k\omega t) \\ &\quad +B_k(\nu)\cos(k\omega t)\Big]\sin(\nu t) \\ &\quad +\sum_{k=0}^{n}\Big[C_k(\nu)\sin(k\omega t) \\ &\quad +D_k(\nu)\cos(k\omega t)\Big]\cos(\nu t) \end{aligned}$$

or with the addition formulas:

$$\begin{aligned} y_s(t) = \frac{1}{2}\sum_{k=0}^{n}\Big[A_k(\nu)\Big(&\cos(k\omega-\nu)t \\ &-\cos(k\omega+\nu)t\Big) \\ +B_k(\nu)\Big(&-\sin(k\omega-\nu)t+\sin(k\omega+\nu)t\Big) \\ +C_k(\nu)\Big(&\sin(k\omega-\nu)t+\sin(k\omega+\nu)t\Big) \\ +D_k(\nu)\Big(&\cos(k\omega-\nu)t-\cos(k\omega+\nu)t\Big)\Big]\ . \end{aligned}$$

If we combine terms with equal frequencies, multiply them with the harmonic functions of the frequencies $k\omega \pm \nu$ and integrate them over the least common period $mT = rT_s$ we get

$$\begin{aligned} J_{1k} &= D_k(\nu) + A_k(\nu) = \\ &= \frac{1}{mT}\int_0^{mT} y_s(t)\cos(k\omega-\nu)t\,\mathrm{d}t \\ J_{2k} &= C_k(\nu) - B_k(\nu) = \\ &= \frac{1}{mT}\int_0^{mT} y_s(t)\sin(k\omega-\nu)t\,\mathrm{d}t \\ J_{3k} &= D_k(\nu) - A_k(\nu) = \\ &= \frac{1}{mT}\int_0^{mT} y_s(t)\cos(k\omega+\nu)t\,\mathrm{d}t \\ J_{4k} &= B_k(\nu) + C_k(\nu) = \\ &= \frac{1}{mT}\int_0^{mT} y_s(t)\sin(k\omega+\nu)t\,\mathrm{d}t\,. \end{aligned} \tag{10}$$

After calculating the integrals in (10) for $k = 0, 1, \ldots, n$ we get for every fixed ν a linear system of equations with the unknown coefficients A_k, B_k, C_k and D_k. For the coefficients we obtain

$$\begin{aligned} A_k &= \frac{J_{1k}-J_{3k}}{2} \qquad & B_k &= \frac{J_{4k}-J_{2k}}{2} \\ D_k &= \frac{J_{1k}+J_{3k}}{2} \qquad & C_k &= \frac{J_{4k}+J_{2k}}{2} \end{aligned} \tag{11}$$

and with (9) we can now calculate $P(\nu, t)$ and $Q(\nu, t)$.

4 Experimental determination

We want to present here three methods for experimental calculation of the PFR. These methods are

- using external input signals,
- using an own system computer and
- using an single input signal.

External input signals

The test configuration is shown in figure 1 with the measuring computer MC and the sampled-data system SDS. According to the formulas (3)-(7) the computer generates the input signal $x_s(t)$ for the time intervall $t \in [0, NT + T_s]$. This is to be done synchronously

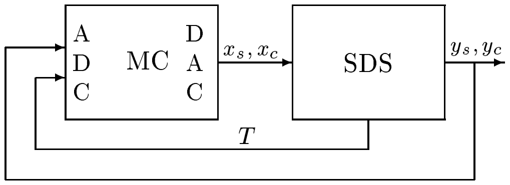

Figure 1: External input signals

to the sampling with the period T_s. During the last period $t \in [NT, NT + T_s]$ we measure the reaction of the system $y_s(t)$ and save the data file $[t, x_s(t), y_s(t)]$. Then we generate the input signal $x_c(t)$, measure the reaction of the system $y_c(t)$ in the same way and save the data file $[t, x_c(t), y_c(t)]$. Now we can calculate $P(\nu, t)$ and $Q(\nu, t)$ with (6) and (7) and generate the PFR with

$$W(j\nu) = P(\nu, t) + jQ(\nu, t), \qquad t \in [0, T_s]\quad .$$

During the sampling period $[NT, NT + T_s]$ it is necessary to get at least 20-50 data of the system reactions $y_s(t)$ and $y_c(t)$. That's why we need a measuring computer and an ADC, which are two dimensions faster then the digital-analogous part of the system. With an measing computer with two synchronized channels it is also possible to get $y_s(t)$ and $y_c(t)$ as parts with phase delay of a single signal for every frequency ν.

Own system computer

If the sampled-data system has its own control computer which is also useable as measuring computer, we can use a test configuration as it is shown in figure 2. Here MCC denotes the measuring and control computer.

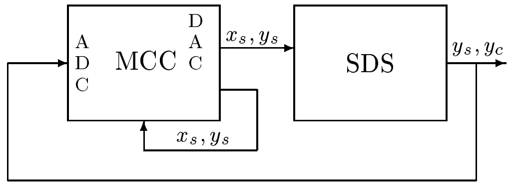

Figure 2: Own system computer

In this case the test program generates the signals $x_s(t)$ and $x_c(t)$, measures the reactions $y_s(t)$ and $y_c(t)$ during the time $[NT, NT + T_s]$ and calculates $P(\nu, t)$ and $Q(\nu, t)$. For this configuration we don't need the synchronization. But as in the first method the capacity of the measuring computer must guarantee to measure at least 20-50 measuring points during the test interval $[NT, NT + T_s]$.

Single input signal

For calculation of the PFR with an alone input signal we use the test configuration shown in figure 3. with the measuring computer MC, the control computer CC and the sampled-data system SDS.

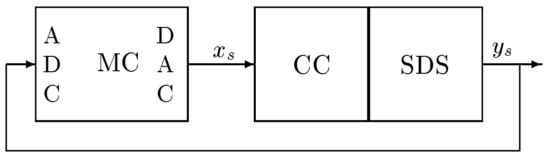

Figure 3: Single input signal

In this case the measuring computer has to measure the signal $y_s(t)$ over the least common period $rT_s = mT$ of the input signal $x_s(t)$ and the system. This means we get the reaction for $t \in [NT, NT+rT_s]$, then we calculate the integrals (10) and form the PFR with (11) and (9).

5 Experimental equipment

It is planned to realize the experimental determination of the PFR with an PC as measuring computer. The PC has a measing card ADC64 with 4 independent A/D-converters every having 100 kHz and 16 Bit resolution, 2 D/A-converter with the same parameters and 4 KByte bidirectional FIFO.

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