DIGITAL CONTROLLER DESIGN BY PARAMETRIC TRANSFER FUNCTIONS (PTF) - COMPARISON WITH OTHER METHODS

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Abstract. The Parametric Transfer Function (PTF) can be regarded as a generalization of the modified z-transformation of sampled-data systems. The delay parameter is extended to the whole time-axis and is denoted by t. The PTF was introduced by L. Zadeh, essential parts were developed by Y. N. Rosenwasser (see [2]). In this article some typical controller design methods will be presented and compared by examples. A direct digital controller design optimizing a continuous performance criterion is compared with standard optimal discrete-time controllers. To performance criterions the continuous-time and discrete-time H_2 norms are applied. The controller design and the simulation of the examples are done in MATLAB/Simulink.

Key Words. Sampled-data system, parametric transfer function, stochastic signals.

1 Introduction

Many control systems consist of continuous plants and digital discrete-time controllers. There are a lot of methods to design a controller for such a hybrid system. In general we have three approaches to this problem. At first we can design an analog controller for the plant and then we discretize this to get an digital controller. The Bilinear Transformation by using Tustin's formula belongs to this class. The second approach is to discretize the plant and make an discrete-time design of the controller. And at third we can directly design a controller for the sampled-data system. The design by Parametric Transfer Functions or the Lifting-technique belongs to this group.

The PTF allows an exact description of linear time-periodic systems in continuous time and needs no approximations. The continuous description allows the evaluation of the continuous error signal, in contrast the normally used discrete-time description takes into account only the error signal at the sampling points. In the case of optimal control this leads to essentially distinct results, even if the sampling period T tends to zero.

2 Properties of the PTF

We have the input signal $x(t)$ and the output signal $y(t)$. Now we assume, that the connection between these two signals is given by an linear operator L

$$y(t) = L[x(t)] \quad .$$

Then we can define the Parametric Transfer Function $W(s,t)$ by

$$W(s,t) = L[e^{st}]e^{-st}$$

with the complex variable s and the time t. It can be shown, that the PTF of time-invariant systems is equal to the ordinary transfer function. For periodic operators with the period T

$$L[x(t+T)] = L[x(t)]$$

the PTF is periodic in t with the same period

$$W(s,t) = W(s,t+T) \quad .$$

Furthermore we get

$$W(s,t) = W_1(s,t) + W_2(s,t)$$

for parallel operators with the PTF $W_1(s,t)$ and $W_2(s,t)$, and

$$W(s,t) = W_1(s)W_2(s,t)$$

for a connection of an linear time-invariant operator with the transfer function $W_1(s)$ and an linear operator with the transfer function $W_2(s,t)$.

If the input signal $x(t)$ of a stable system with the PTF $W(s,t)$ is an disturbance signal with the spectral density $S_x(s)$, the output would be an instationary process $y(t)$ with the time-depending variance

$$\nu_y(t) = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} W(s,t) S_x(s) W(-s,t)\, ds.$$

3 Design algorithm

With the PTF we can solve the following control problem: Which controller $C(z)$ minimizes the average variance of the output $y(t)$, if the transfer function $F(s)$ of the plant, the spectral density $S_w(s)$ of the disturbance signal $w(t)$ and the sampling period T are given. Let $F(s)$ be given by

$$F(s) = \sum_{i=1}^{n} \frac{\alpha_i}{s - s_i} \quad .$$

Then we define

$$d_F(z^{-1}) = \prod_{i=1}^{n} \left(z^{-1} - e^{-s_i T}\right) \quad .$$

In the same way we have

$$\begin{aligned} H(s) &= h_0 + \sum_{i=1}^{m} \frac{\beta_i}{s - r_i} \\ p_H(z^{-1}) &= \prod \left(z^{-1} - e^{r_i T}\right) \\ S_w(s) &= \sum_{i=1}^{l} \frac{\gamma_i}{s - q_i} - \sum_{i=1}^{l} \frac{\gamma_i}{s + q_i} \\ d_w(z^{-1}) &= \prod_{i_1}^{l} \left(z^{-1} - e^{-q_i T}\right) \quad . \end{aligned}$$

With the indices '+' and '-' we denote the stable and the the unstable part of a polynom. In [4] the following algorithm is presented for solving the control problem:

1. Perform the factorization

   $$A(z^{-1}) = K(z^{-1}) K(z)$$

   with all zeros and poles of $K(z)$ inside the unit circle. Here we have

   $$K(z^{-1}) = \frac{f d_H^+ p_H^-}{d_w}$$

   with a certain polynom $f(z^{-1})$.

2. Construct the function

   $$L_1(z^{-1}) = \frac{g_1}{z^{-\nu} d_F d_w d_H^- \hat{f}}$$

   with the polynom

   $$\hat{f} = f(z) z^{-\deg f}$$

   and separate it into

   $$L_1(z^{-1}) = \frac{\vartheta_1}{d_F d_w} + \frac{\pi_1}{z^{-\nu} d_H^- \hat{f}} \quad .$$

3. Make the separation

   $$\frac{K}{z^{-1} n(z^{-1}) d_F d_H} = \frac{\vartheta_2}{d_H^-} + \frac{\pi_2}{z^{-1} n d_F d_w} \quad .$$

4. Find the function $\vartheta(z^{-1})$ by

   $$\frac{\vartheta}{d_F d_H^- d_w} = \frac{\vartheta_1}{d_F d_w} + \frac{\vartheta_2}{d_H^-} \quad .$$

5. With the polynomials

   $$\begin{aligned} n_c(z^{-1}) &= d_H^+ \vartheta \\ d_c(z^{-1}) &= \frac{p_H^- f - z^{-1} n \vartheta}{d_F d_H^-} \end{aligned}$$

   we get the discrete transfer function of the optimal controller

   $$C(z^{-1}) = \frac{n_c(z^{-1})}{d_c(z^{-1})} \quad .$$

4 Numerical results

In our examples we regard a ship under stochastic disturbances by sea and wind. As linear model for the ship we have

$$F(s) = \frac{0.051}{25s^2 + s} \quad .$$

For the stochastic disturbance we assume the spectral density

$$S_w(s) = \frac{0.0757}{s^4 + 2.489 s^2 + 1.848}$$

and we assume a sampling time of $T = 1$.

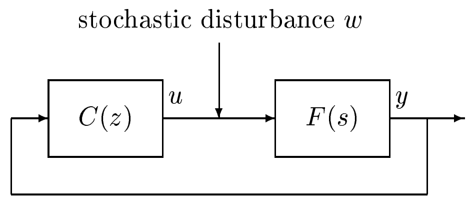

Figure 1: control system

So we have the above control system which we want to control with different controllers.

With the use of the PTF in [4] the controller

$$C(z) = \frac{76.125z^3 - 120.426z^2 + 93.372z - 43.562}{z^3 + 0.705z^2 + 0.117z + 0.0458}$$

is designed as the optimal digital controller for the given problem. This controller is designed to minimize the course-deviation with respect to the rudder angle

$$E = \nu_y + \rho\nu_u$$

with the variances ν_y, ν_u of course und rudder. The performance criterion

$$E_{t_1,t_2} = \int_{t_1}^{t_2} y(t)^2 + \rho u(t)^2 \, dt \tag{1}$$

of some time-intervals are presented in table 1 and is compared to the discrete divergence criterion

$$S_{k_1,k_2} = \sum_{k=k_1}^{k_2} T \cdot \left(y(kT)^2 + \rho u(kT)^2\right)$$

Typical results of simulations are shown in the figures 2 and 3, where the course $y(t)$, the rudder angle $u(t)$ and the disturbance $w(t)$ are to be seen.

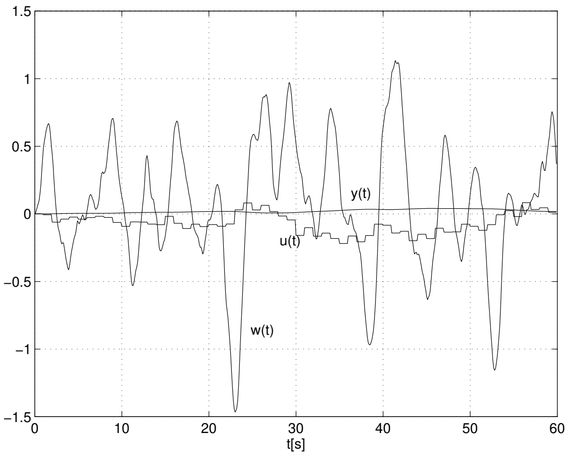

Figure 2: PTF-controller

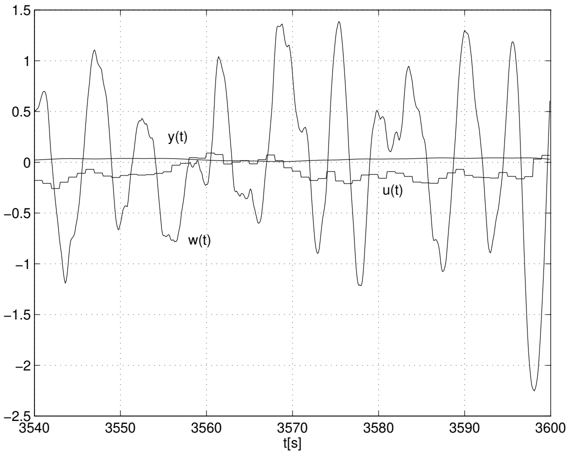

Figure 3: PTF-controller

A common way to design a controller for a system with stochastic disturbances is to design the standard minimal variance controller. This is not possible for our system, because there is an I-part in the plant. If we design the PID-controller with

$t_1 ... t_2$	E_{t_1,t_2}	S_{k_1,k_2}
0...120	0.3384	0.3396
120...240	0.2172	0.2210
240...360	0.3067	0.3107
360...480	0.5622	0.5633
480...600	0.5177	0.5213
600...720	0.3118	0.3122
720...840	0.8406	0.8411
840...960	0.3043	0.3236
960...1080	0.4624	0.4666
1080...1200	0.2568	0.2651
1200...1320	0.4393	0.4406
1320...1440	0.3928	0.3931
1440...1560	0.4231	0.4351
1560...1680	0.3587	0.3613
1680...1800	0.2779	0.2783
1800...1920	0.2972	0.2992
1920...2040	0.3651	0.3667
2040...2160	0.5704	0.5730
2160...2280	0.6463	0.6539
2280...2400	0.4467	0.4550
2400...2520	0.3790	0.3811
2520...2640	0.2743	0.2780
2640...2760	0.4363	0.4528
2760...2880	0.8024	0.8077
2880...3000	0.3485	0.3504
3000...3120	0.2784	0.2866
3120...3240	0.4677	0.4694
3240...3360	0.1928	0.1935
3360...3480	0.1285	0.1287
3480...3600	0.2383	0.2399

Table 1: Deviation of the PTF-controller

the rules of Ziegler/Nichols/Takahashi, we get

$$C(z) = \frac{12.6697z^2 - 23.5113z + 11.7647}{z^2 - z}$$

as digital controller. But this controller works not stable, because the stochastic disturbance results in increasing oscillations of the system, as it is shown in figure 4.

That's why we want to compare our controller to a controller, which we design by pole-placement. By this method we get

$$C(z) = \frac{3.7087z^2 - 3.5218z}{z^2 - 1.0005z + 0.0007}$$

as an stable controller for the given system. Table 2 shows the continuous and discrete divergence for this controller.

With the results in table 1 and table 2 it is obvious, that the PTF-designed controller gives much better performance as the controller, designed by pole-placement.

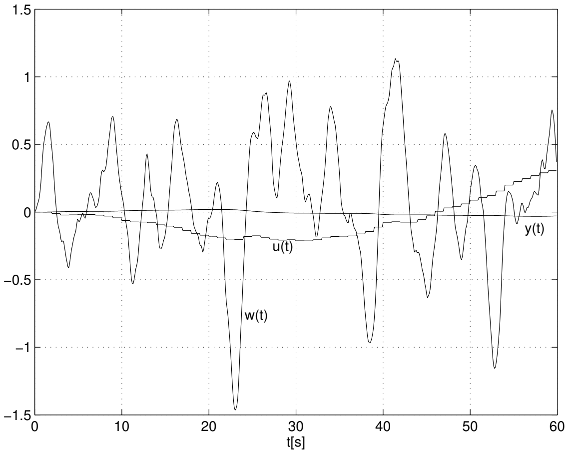

Figure 4: PID-controller

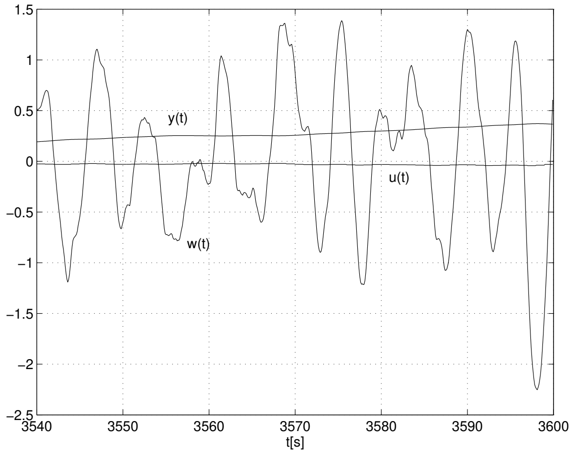

Figure 5: controller with pole-placement

$t_1...t_2$	E_{t_1,t_2}	S_{k_1,k_2}
0...120	0.825	0.834
120...240	4.201	4.215
240...360	1.854	1.859
360...480	4.759	4.825
480...600	33.817	34.081
600...720	45.002	45.273
720...840	7.809	7.973
840...960	5.143	5.194
960...1080	10.133	10.159
1080...1200	9.436	9.518
1200...1320	3.383	3.439
1320...1440	0.905	0.909
1440...1560	1.707	1.712
1560...1680	0.141	0.142
1680...1800	0.738	0.745
1800...1920	10.680	10.768
1920...2040	6.766	6.854
2040...2160	1.342	1.351
2160...2280	4.403	4.406
2280...2400	5.275	5.298
2400...2520	7.932	8.026
2520...2640	6.473	6.547
2640...2760	0.389	0.393
2760...2880	16.508	16.563
2880...3000	3.657	3.736
3000...3120	7.002	7.047
3120...3240	3.026	3.046
3240...3360	1.005	1.010
3360...3480	0.664	0.674
3480...3600	5.978	6.049

Table 2: Deviation of the controller with pole-placement

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